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$$\frac{1}{5} + \frac{r}{500} \text{ loss, or } \frac{1}{5} + \frac{r}{500} = \frac{1.5\%}{500}, \quad \frac{r}{500} = \frac{5\%}{500}, \text{ or } r=50.$$

Do you call this an Arithmetical solution? Instead of r , suppose you write "same"? Pure Algebra: $1 + \frac{r}{100} = \frac{5}{4} \left(1 + \frac{r}{100} - \frac{1.5\%}{100} \right)$. Then $r=50$.

[NOTE.—According to our definition of an algebraical solution, No. 5, Vol. V., page 139, of the MONTHLY, Professor Bradbury's solution is algebraic. It is immaterial as to what sort of a character is used to represent the quantity sought. It may be a letter, a character of any kind, a word, or several words.

The definition referred to, viz.: Any solution in which the result sought is represented by some character, which character is operated upon until the condition or conditions of the problem are fulfilled which condition or conditions are stated in the form of an equation from which the numerical value of the character is determined, is an algebraic solution,—has received the sanction of some of the best mathematicians in this country. Editor F.]

II. Solution by J. OWEN MAHONEY, B. E., Mc., Professor of Mathematics and Science, Carthage Graded and High School, Carthage, Tex.; J. W. YOUNG, Columbus, O.; ELMER SCHUYLER, High Bridge, N. J.; WALTER H. DRANE, Graduate Student, Harvard University, 65 Hammond Street, Cambridge, Mass.; and the PROPOSER.

Since 100% of the purchasing price is paid for the goods in the first instance, it is easily seen that

$$\begin{aligned} &\text{the \% gain} - 20\% : 120 :: \text{the \% gain} - 25\% : 100, \\ &\text{or the \% gain} - 20\% : 5\% :: 120 : 20 \\ &\text{or the \% gain} - 20\% : 5\% :: 6 : 1. \end{aligned}$$

$$\therefore \text{The \% gain} = 50.$$

III. Solution by B. F. FINKEL, A.M., M.Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

1. Let 100% = actual cost, then
2. 120% = supposed cost.
3. Let 100% = selling price. [The italics are used for distinction].
4. 100% - 100% = actual gain, and
5. 100% - 120% = supposed gain.

$$6. \quad \frac{100\% - 100\%}{100\%} - \frac{100\% - 120\%}{120\%} = \frac{1}{4}, \text{ or } \frac{100\% - 100\%}{100\%} - \frac{83\frac{1}{3}\% - 100\%}{100\%} = \frac{1}{4},$$

$$\text{or } \frac{16\frac{2}{3}\%}{100\%} = \frac{1}{4}, \text{ whence}$$

7. $16\frac{2}{3}\% = 25\%$; $1\% = 1.5\%$; $100\% = 150\%$, selling price in terms of cost price.
8. $\therefore 150\% - 100\% = 50\%$, the gain %.

Also solved by G. B. M. ZERR and B. F. YANNEY.

105. Proposed by ALOIS F. KOVARIK, Instructor in Mathematics and Physics, Decorah Institute, Decorah, Iowa.

A teacher looks at his watch when leaving school at noon. When he comes back he finds that the hour hand and the minute hand have just changed places (that they had when he left the school). What time was it when he left, and what time when he came back to school? (Solve by Arithmetic.)

I. Solution by JOHN M. ARNOLD, Crompton, R. I.

During the time that the teacher was away from the school, the distance traveled by the minute hand added to the distance traveled by the hour hand,

would make the whole circle of the dial. As the minute hand moves twelve times as fast as the hour hand the spaces passed over would be $\frac{1}{3}$ and $\frac{1}{12}$ of a revolution, respectively.

$\frac{1}{3}$ of a revolution of the minute hand equals $55\frac{5}{13}$ minutes, the time the teacher was away. When the teacher left, the distance between the hands was $\frac{1}{2}$ of the distance of the minute hand from the zero point. Hence $\frac{1}{3} \times \frac{1}{12} = \frac{1}{48}$. $= 5\frac{5}{13}$ minutes past twelve, the time when he left. Add the time that he was away, $5\frac{5}{13} + 55\frac{5}{13} = 60\frac{10}{13}$ or $\frac{60}{13}$ of a minute past one, the time when he returned.

II. Solution by BENJAMIN F. PANNEY, A. M., Professor of Mathematics, Mount Union College, Alliance, O.

We first solve the general problem : At what times are the positions of the hands of a watch interchanged ?

It is plain that there are some positions not interchangeable, as, for instance, minute-hand at 6 and hour-hand midway between any two consecutive numbers.

It is evident, also, that in the case of any possible position of interchange, in going from one position to the interchanged one, the two hands must together travel over $60n$ minute spaces, n being an integer.

Then, since the minute-hand travels 12 times as fast as the hour-hand, $\frac{1}{13}$ of the distance traversed by both, or $60n/13$ minute spaces, must be the distance traveled by the hour hand in going from one position of interchange to the other, and $\frac{12 \cdot 60n}{13}$ minute-spaces, the minute-hand's distance. Furthermore, $60n/13$ minute-spaces is the distance in any case of interchange, from hour-hand to minute hand, always reckoned clockwise.

We now have the relative positions of the hands with respect to each other. We next proceed to find the times of these relative and interchanged positions. Suppose the hands at 12. In order that they shall be $60n/13$ minute-spaces apart, the minute-hand must travel over $\frac{12 \cdot 60n}{11 \cdot 13}$ minute-spaces. This, the number of minute-spaces past 12, gives the time of the first of any two interchanged positions, and $\frac{12 \cdot 60n}{11 \cdot 13} + \frac{12 \cdot 60n}{13}$ is the time of the corresponding second position.

Now starting, say, with 12 o'clock noon, and substituting for n in succession 1, 2, 3, 143, which completes a cycle, we shall find, omitting the cases in which the hands are together, 132 different answers to the general problem. We give a few :

$5\frac{5}{13}$ minutes past 12 noon, and $\frac{60}{13}$ of a minute past 1 P. M.
 $10\frac{10}{13}$ " " " " " $\frac{120}{13}$ " " " 2 P. M.

.....
 $\frac{60}{13}$ of a minute past 1 P. M., and $5\frac{5}{13}$ minutes past 12 A. M.

.....
 $59\frac{8}{13}$ minutes past 10 P. M., and $54\frac{3}{13}$ minutes past 11 P. M.

The first two given are possible answers to the special case in hand.

III. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; B. F. SINE, Principal Capon Bridge Normal School; Capon Bridge, W. Va.; J. D. CRAIG, Frankfort, N. J.; and MARTIN SPINX, Wilmington, O.

Since the two hands had precisely changed positions, they together had passed over all the spaces on the dial-face ; but, as the minute-hand always goes through 60 spaces while the hour-hand goes through 5, both go through 65.

$$\therefore 65:5=60:4\frac{8}{3}.$$

$\therefore 4\frac{8}{3}$ spaces is the number of spaces passed over by the hour-hand. This is also the distance the minute-hand was in advance of the hour-hand in the first position.

Since the time he left at noon was after 12 o'clock and since the minute-hand always gains 55 minutes in 60 minutes, to gain $4\frac{8}{3}$ minutes we have $55:60::4\frac{8}{3}:5\frac{5}{4}\frac{5}{3}$.

\therefore The time was $5\frac{5}{4}\frac{5}{3}$ minutes after 12 o'clock. In the second position, the hour-hand was $4\frac{8}{3}$ minutes in advance of the minute-hand. $5\frac{5}{4}\frac{5}{3}-4\frac{8}{3}=1\frac{6}{4}\frac{9}{3}$ minutes.

\therefore The time was $1\frac{6}{4}\frac{9}{3}$ minutes after 1 o'clock.

\therefore He left at 5 minutes $2\frac{1}{4}\frac{4}{3}$ seconds after 12 o'clock, and returned at $25\frac{3}{4}\frac{5}{3}$ seconds after 1 o'clock.

Also solved by W. F. BRADBURY, J. W. YOUNG, WALTER H. DRANE, ELMER SCHUYLER, and ALOIS F. KOVARIK

ALGEBRA.

89. Proposed by G. A. MILLER, Ph. D., Instructor in Mathematics, Cornell University, Ithica, N. Y.

$$\begin{aligned} \text{Solve by quadratics,} \quad x^2 + y &= 7 \dots\dots (1), \\ x + y^2 &= 11 \dots\dots (2). \end{aligned}$$

XI. Solution by W. A. HARSHBARGER, A. M., Professor of Mathematics, Washburn College, Topeka, Kas.

$$y^2 + x = 11 \dots\dots (1), \quad y + x^2 = 7 \dots\dots (2).$$

$$(1) - (2) \quad (y^2 - x^2) - (y - x) = 4 \dots\dots (3).$$

$$\text{Put } (y + x) = a, \text{ and } (y - x) = b.$$

$$\text{Then by substituting in (3), } ab - b = 4 \dots\dots (4).$$

$$\therefore a^2 b^2 = 16 + 8b + b^2 \dots\dots (5).$$

$$\text{Subtract, } 10ab = 40 + 10b \dots$$

$$\therefore a^2 b^2 - 10ab = -24 - 2b + b^2 \dots\dots (6), \text{ and}$$

$$a^2 b^2 - 10ab + 25 = 1 - 2b + b^2 \dots\dots (7).$$

$$\therefore ab - 5 = 1 - b, \quad ab + b = 6 \dots\dots (8). \quad (4) + (8), \quad ab = 5; \quad (4) - (8), \quad b = 1.$$

$$\therefore a = 5. \quad \therefore y + x = 5, \text{ and } y - x = 1. \quad \therefore x = 2, \text{ and } y = 3.$$

[NOTE. Professor Harshbarger says the above solution appeared in one of the scientific journals a few years ago, but he has forgotten the name of the author.]